The incompressible Navier-Stokes equations: revisiting a classical formulation with modern numerical techniques

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The primitive-variable formulation of the incompressible Navier-Stokes equations is the standard framework for numerical simulations of incompressible flows. However, the pressure-velocity coupling remains its most significant computational challenge. The classical streamfunction-vorticity formulation, introduced in 1933, offers an attractive alternative in this regard as it eliminates the pressure variable and, consequently, the need for specialised pressure-velocity coupling algorithms. As a result, it was widely adopted for two-dimensional incompressible flow simulations between the 1960s and the 1990s. Nevertheless, the mathematical derivation and numerical treatment of physically meaningful vorticity boundary conditions, especially in domains with complex geometries, is challenging. Alongside the rapid development of robust pressure-velocity coupling algorithms (such as SIMPLE, PISO, and projection methods), and the increasing demand for three-dimensional simulations, primitive-variable formulations have become the dominant computational framework since the 1990s.

Recent advances in very high-order accurate numerical methods, particularly techniques for the accurate treatment of boundary conditions on complex geometries, provide an opportunity to revisit the classical streamfunction-vorticity formulation from a modern perspective. In this talk, we present a comparative study of primitive-variable and streamfunction-vorticity formulations within a very high-order accurate finite volume framework. Particular emphasis is placed on the mathematical derivation and numerical treatment of streamfunction and vorticity boundary conditions on arbitrary curved geometries using accurate boundary discretisation techniques. The accuracy and performance of both formulations are assessed through several two-dimensional benchmark problems, highlighting their respective strengths and limitations in terms of numerical accuracy and computational efficiency.

Can recent advances in high-order accurate numerical methods make a classical Navier-Stokes formulation competitive once again? This talk is intended for non-specialists, so everybody is welcome.

This is a joint work with Stéphane Clain (University of Coimbra), Gaspar J. Machado (University of Minho), and João M. Nóbrega (University of Minho).